

Political Institutions and Sorting in a Tiebout Model

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We construct a computational model of Tiebout competition and show that political institutions differ in their ability to sort citizens effectively. In particular, we find that certain types of institutions—those that become more “politically unstable” as citizen heterogeneity increases—perform relatively poorly given a single jurisdiction, yet these same institutions perform relatively well when there are multiple jurisdictions. We provide an explanation for this phenomenon which draws upon simulated annealing, a discrete nonlinear search algorithm. (JEL H21, H41)

Political or economic institutions are often evaluated by their ability to structure micro-level incentives to be in agreement with macro-level goals (Thomas C. Schelling, 1978). In many important situations, micro-level incentives are consistent with multiple equilibria (Robert Axelrod and P. Scott Bennett, 1993; Arthur DeVany, 1994), and one role of institutions may be to steer agents in the direction of configurations of highest aggregate utility, or at least bias outcomes toward better configurations. In this paper, we compare the performance of three common political institutions in a model of Tiebout competition. We examine the ability of alternative political institutions to induce better sorting of citizens among competing local jurisdictions, where better sorting means higher aggregate utility among citizens. While the focus of this paper is on Tiebout models, the underlying process we analyze—the decentralized sorting of interacting individuals—is of much broader interest, playing a role in labor markets, industrial location, and coalition formation.

In a Tiebout model, local jurisdictions compete for citizens by offering bundles of public goods. Citizens then sort themselves among jurisdictions according to their preferences. Charles M. Tiebout's (1956) original hypothesis challenged Paul Samuelson's (1954) conjecture that public goods could not be allocated efficiently. The Tiebout hypothesis has since been extended to include additional propositions. Prominent among them is that competition, as a result of enforcing efficiency, renders local politics unimportant: a jurisdiction able to attract and retain citizens cannot waste resources (William H. Hoyt, 1990). This argument does not, however, preclude the possibility that political institutions may differ in their ability to sort citizens according to preferences. In fact, we find that important differences do exist among institutions.

To analyze institutional differences, we construct a computational Tiebout model. Using the model we directly investigate the dynamics of agent sorting in the context of institutional arrangements that model existing political structures. The focus on dynamics (as opposed to final equilibrium states), and the use of institutions that resemble those actually employed (versus a fictional, profit-maximizing entrepreneur), distinguish our work from prior research on the Tiebout hypothesis. The computational approach complements existing efforts by enabling us to study questions that are currently unassailable if we require closed-form, equilibrium solutions. We use computation as a way to guide and test our intuitions

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as to how these more complex systems operate.

We find that significant differences arise in the performance of different political institutions. In the model, each political jurisdiction confronts a finite number of issues, which may be considered as either local policies or public projects. Citizens have linearly separable preferences over these issues, and they choose to locate in the jurisdiction offering the most attractive set of policies on the issues. Jurisdictions decide policies using one of three possible types of political institutions: democratic referenda, direct competition, or proportional representation. Like standard Tiebout models, we find that, regardless of the political institution employed, aggregate utility increases with the number of jurisdictions. However, quite unexpectedly, we also find that institutions that perform relatively poorly in the presence of a single jurisdiction perform relatively well under multiple jurisdictions. In fact, institutions characterized by higher degrees of "political instability" appear to induce better sorting when multiple jurisdictions are present.

We offer an explanation for this phenomenon based on insights from the literature on nonlinear search algorithms, specifically models of simulated annealing. The analysis suggests that in order for political instability to improve outcomes in a multiple-jurisdiction environment, the degree of instability must be positively correlated with the heterogeneity of citizens' preferences. Such a correlation allows the system to break out of inferior configurations and eventually stabilize on superior ones.

The potential benefit of policy instability on aggregate welfare is suggested by the following simple example. Consider a situation in which two towns must choose between either allowing alcohol sales or being dry. There are three citizens in each town, two of whom favor having alcohol over being dry. Under majority rule on the issue, both towns allow alcohol, and no citizen wants to move. Clearly majority rule prevents the symmetry from ever being broken in the two towns. However, suppose that one of the towns happens to become dry. When this occurs, citizens immediately resort, resulting in one town of four citizens that

allows alcohol and one of two citizens that does not. The policy instability has broken the initial symmetry and leads to the creation of an equilibrium configuration of the citizens with a higher level of aggregate welfare.

Economic processes are often governed by the decentralized sorting of agents, such as buyers seeking sellers, workers looking for employers, firms finding locations, and individuals forming coalitions. The existence of multiple equilibria under decentralized sorting suggests that the dynamics implied by alternative institutional arrangements may influence the final configuration of agents. We find that certain institutions create conditions under which the sorting of agents naturally tends towards those equilibria with superior configurations.

I. The Tiebout Hypothesis

This section briefly summarizes research concerning Tiebout's model and indicates how our work departs from previous research. Tiebout's (1956) hypothesis is that local public goods might be efficiently allocated if agents can freely choose among alternative jurisdictions actively seeking citizens. The Tiebout hypothesis has been investigated in depth both theoretically and empirically.

The theoretical literature has uncovered some problems with Tiebout's argument, and many theorists have clarified and extended Tiebout's original model. Truman F. Bewley (1981) showed through a series of examples that regardless of whether the costs of local public goods are independent of, or proportional to, the size of the local population, there exist economies with equilibria that are not Pareto optimal. In other words, the pure Tiebout hypothesis does not hold. The existence of examples with suboptimal equilibria is not sufficient to dispose with Tiebout's theory. More troubling is Bewley's result that the Tiebout hypothesis can be expected to hold only when the number of jurisdictions is at least as large as the number of citizens—a rather unlikely situation. By relaxing some of Bewley's assumptions, efficient equilibria can be generated more easily. One approach employs ε -equilibrium as a proxy for moving costs [see Myrna H. Wooders (1994) for an

overview]. Other work (for example, Hideo Konishi, 1996) has generated equilibrium properties by assuming particular preferences or a single political institution. There are also models in which public goods suffer from differential crowding (Eitan Berglas, 1976; John P. Conley and Wooders, 1997), and models with divisible land (Susan Rose-Ackerman, 1979). This paper departs from the existing theoretical research by comparing the performance of different political institutions in a Tiebout setting.

The empirical literature on Tiebout [see Keith Dowding et al. (1994) for a recent survey] has focused on a variety of questions, including whether competition among jurisdictions leads to efficiency, the impact of bundles of local public goods on jurisdictional choice, and the role of political institutions in public goods provision. Empirical findings concerning the first question are inconclusive and appear to depend upon whether the local jurisdictions are single or multipurpose (Jeffrey S. Zax, 1989).¹ The analysis of how bundles of public goods enter into jurisdictional choice fall into macro- and micro-level studies. In a number of cases using aggregate data, fiscal differences among jurisdictions have been shown to have a significant effect on migration (Andrew Reschovsky, 1979).² These studies find that fiscal factors play a larger role in pushing people out of jurisdictions than in pulling people into new jurisdictions, which is not surprising given the asymmetric information (William F. Fox et al., 1989). Other studies rely on micro-level survey data to examine the link between public services and goods, and jurisdictional choice. The findings in these studies are less conclusive, although school quality and tax rates appear as significant in several studies (E. B. Sharp, 1984; Stephen L. Percy and Brett W. Hawkins, 1992; Percy, 1993). Finally, welfare

policies across states in the United States do seem to influence migration patterns of the poor (Paul Peterson and Mark Rom, 1989). The question of how political institutions influence public goods provision is perhaps the most relevant to our analysis. Unfortunately, with the notable exception of Thomas Romer and Howard Rosenthal's (1979) work on school spending referenda, this area has not been as widely investigated. Romer and Rosenthal find that the reversion level (what the policy will be if the referendum fails) influences the outcomes of referenda.

Our model departs from previous approaches in several important respects. First, with a few exceptions, our primary interest in comparing the performance of political institutions has been largely neglected in the Tiebout literature. A typical Tiebout model takes the political institution, usually majority rule, as constant. Here we vary institutions and measure performance, an approach more consistent with the literature on mechanism design. Second, aside from an example used to demonstrate the annealing phenomenon, we do not explicitly compare equilibria. This decision stems partly from a desire to avoid oversimplifying an already stark model by making the restrictive assumptions necessary to guarantee the existence of equilibria. It is also consistent with our emphasis on the dynamic nature of agent migration, political responsiveness, and the production of public goods. People may continue relocating in response to changing policies, while local policies shift, in turn, to changes in constituencies. The system of policy choices by governments and jurisdictional choices by citizens can be viewed as a complex adaptive system in which movements and policies are determined by the preferences of citizens and the existing political institutions. Some institutions may create systems which settle quickly into equilibrium while others may never equilibrate. Finally, our assumptions of how jurisdictions make decisions differ from much of the previous literature. Rather than model political parties as fully informed and optimizing, we rely on an adaptive party model in which parties gather information by taking polls, and decide on platforms by using simple search heuristics (John H. Holland and Miller, 1991; Kollman et al., 1992).

¹ Zax shows that the lowest levels of tax appears to be paid when there is high fragmentation among multipurpose jurisdictions, low fragmentation of single-purpose jurisdictions, and decentralized governments.

² Richard J. Cebula (1974) shows that welfare payments had a significant effect on interstate migration in the United States from 1965–1970.

II. The Model

We construct a simple version of a Tiebout world. Agents (citizens) seek their favorite location from among a fixed set of jurisdictions each of which offers a particular platform of local public goods and policies. Each jurisdiction's platform is then determined by a political institution that aggregates the preferences of the agents currently residing in the jurisdiction. Once new platforms are determined, agents are allowed to move, and the above process is iterated. Agents do not take into account the effects of their own movements on future policies.

We assume a set of N_a agents, each of whom must reside in one of N_j possible jurisdictions. Within any jurisdiction, the local government is obliged to decide positions on a set of N_i local public issues.³ All jurisdictions must take positions on each of the N_i issues. For simplicity we assume that all such positions are binary. Examples of such issues include the presence (or absence) of a public good (say a community swimming pool) or policy (like smoking in public buildings).⁴ Let $p_{ji} \in \{Yes, No\}$ give the position of jurisdiction j on issue i , and let a platform, $\mathbf{P}_j \in \{Yes, No\}^{N_i}$, denote the vector of decisions across all N_i issues in jurisdiction j . Finally, define a configuration as a mapping of agents to jurisdictions.

Agents have linearly separable preferences on the issues and their per unit value for each issue lies in the interval $[-400/N_i, 400/N_i]$ distributed uniformly. Let ν_{ai} give agent a 's utility for issue i . If the project is undertaken, then the agent obtains a utility of ν_{ai} , and if the project is not undertaken, then the agent obtains no utility. Thus, agent a 's utility from \mathbf{P}_j is given by

$$u_a(\mathbf{P}_j) = \sum_{i=1}^{N_i} \nu_{ai} \cdot \delta(p_{ji}),$$

where $\delta(Yes) = 1$ and $\delta(No) = 0$. A straightforward calculation verifies that the expected value to an agent of an arbitrary platform equals zero, and the expected value of her optimal platform equals 100.

A. Political Institutions

We compare three political institutions: *democratic referenda*, *direct competition*, and *proportional representation*.⁵ The simplified versions of these institutions used in the model attempt to capture the relevant features of each institution without overly complicating the analysis.

Democratic Referenda

We model democratic referenda as simple majority rule on each issue.⁶ Our assumption that there are no external effects between projects implies that sincere voting is a dominant strategy for all agents within a given election. The outcome of democratic referenda is the *median platform*, \mathbf{P}_j^m , for the jurisdiction, where $p_{ji}^m = Yes$ if the number of agents in j with $\nu_{ai} > 0$ exceeds the number of agents in j with $\nu_{ai} < 0$; otherwise $p_{ji}^m = No$. The median platform maximizes aggregate utility in jurisdiction j if and only if on every issue the mean and median agent values have identical signs.

⁵ Democratic referenda are used by many cities and states in the United States to decide particular policy issues. National referenda were used by Norway and Great Britain to decide whether or not to enter the European Union, and Australia and several smaller European countries (including Switzerland, Denmark, and Ireland) regularly employ this institution on a national basis. Direct competition (winner-take-all plurality voting) is used by most major cities and all states in the United States to elect executives (mayors and governors). Many countries, including Canada, South Africa, and the United Kingdom, also use direct competition to elect local and national leaders. Proportional representation is widely used by most of the world's democracies. Proportional systems vary greatly, but they all have the common feature of attempting to link legislative representation to proportionality of the popular vote.

⁶ In the case of a tie, we assume that the decision is *No*.

³ For convenience each agent, jurisdiction, and issue is given an integer label on $\{1, \dots, N_k\}$, where $k = a, j$, and i respectively.

⁴ Although real-valued issues are often used in Tiebout models (for example, jurisdictions differ in the amounts of public goods or tax rates they offer), using binary issues simplifies the modeling of the requisite political institutions. Experiments with real-valued versions of the model under direct competition and proportional representation indicate that our results appear robust to this change.

Generally speaking, democratic referenda adopt platforms of relatively high aggregate utility given a group of agents.

Democratic referenda are relatively stable, since the implied platform is unique and individual agents migrating into or out of a jurisdiction rarely change the platform. Suppose that a single agent moving into a jurisdiction prefers *Yes* on issue one. The only way that she can change the decision on this issue is if she is pivotal; that is, if prior to her moving, an equal number of agents had positive and negative values on the issue.

Direct Competition

We model direct competition as winner-take-all plurality voting among parties advocating different platforms. Under direct competition, each agent votes for the party proposing the platform that yields her the highest utility, and the winning party wholly implements its platform in the jurisdiction.

Direct competition may not be as stable as democratic referenda. Even with the linearly separable preferences considered here, policy predictions under direct competition cannot be guaranteed to produce unique solutions without severe restrictions on preferences (Charles R. Plott, 1967)—cycling is endemic to plurality rule. Given the potential for cycling, various alternative solutions have been proposed, including the top-cycle set (Richard D. McKelvey, 1976), the uncovered set (McKelvey, 1986), and the minmax set (Gerald H. Kramer, 1977). Unfortunately, even these solutions often imply disparate platforms or fail to narrow the potential set of platforms.⁷ The absence of a unique equilibrium in direct competition for large classes of preferences has led some to call into question the effectiveness of democratic decision-making (William Riker, 1982). For our purposes, predicting an exact platform is less important than knowing the bounds on the set from which platforms will be chosen. Here we rely on an adaptive party model (discussed below) in which

parties with incomplete information choose platforms using simple search heuristics. The platforms that emerge in the adaptive model often correspond to the equilibria generated by the mathematical solutions. Evidence from our experiments suggests that when a single platform is predicted using mathematical techniques, the adaptive parties tend to find it within a few elections, while in situations where the formal predictions are set valued, the adaptive parties tend to advocate platforms from within or near the proposed solution set.

Proportional Representation

Under proportional representation each agent votes for a single party, and each party receives a number of seats in a decision-making body proportional to its popular vote. In our model we assume no distortion between the percentage of the vote that a party receives and the percentage of seats it obtains in the legislature. In the model, once each party is allocated seats, the final decision on each issue is the weighted (by popular vote) sum of each party's platform position on each issue. In essence, each party sincerely votes in a series of referenda across the issues. For example, if a party advocating the platform (*Yes, No, Yes*) receives 18 percent of the popular vote, then that party would vote *Yes, No, and Yes* on the first, second, and third issues respectively, and these three votes would each be assigned a weight of 0.18. The final decision on each issue equals *Yes* if the total weight of all parties in the jurisdiction advocating *Yes* exceeds 0.5 and *No* otherwise.⁸ We assume that agents vote sincerely, although we discuss the issue of strategic voting at the end of the paper.⁹

Although not reported here, we also experimented with a version of proportional

⁸ These assumptions do not preclude a party that receives a small percentage of the vote from having substantial power.

⁹ In elections with more than two parties, a voter may have an incentive to act strategically. Suppose that a voter with ideal platform (*Yes, No, No*) is given a choice between the platforms (*No, No, No*) and (*Yes, Yes, Yes*), and given this choice she would pick (*No, No, No*). If the weighted vote on *Yes* for the first issue is close to 0.5 and significantly below 0.5 on the second and third issues, then

⁷ For example, one can generate situations in which the top-cycle set encompasses the entire platform space, while the uncovered set—a refinement of the top-cycle set—contains every platform except the median one.

representation that relied on a Borda-like weighting scheme. In this version, an agent was allowed to allocate $m - 1$ votes to her favorite party, $m - 2$ to her next favorite, and so on, where m is equal to the number of parties on the ballot. The number of seats used in the final weighting was then set to the proportion of votes received by each party. We found that such a system often leads to superior outcomes, most likely due to its ability to convey intensity of preferences among the electorate. Nonetheless, given the relative lack of real world examples of such a system (Australia and Ireland do rely somewhat on related systems), and our desire to simplify the presentation, we do not report the results.

Under direct competition and proportional representation there is the potential for more than two parties. Note that proportional representation and direct competition lead to identical outcomes when there are only two parties in the system, since under proportional representation the party with the majority of votes will be able to impose its entire platform. As the number of parties increases above two, the platform used in a jurisdiction may differ between the two institutions if no party receives more than half the vote, since platforms other than the winning party's will affect the final policy decisions under proportional representation. In our model we assume that the number of parties is given exogenously. While party formation is of theoretical and practical importance, we leave the analysis of this process to future research.

B. Adaptive Parties

As discussed above, mathematical models of party behavior may not adequately restrict the set of potential platforms. To overcome this problem, we use a model developed by Kollman et al. (1992) in which parties adapt platforms using search heuristics applied to polling information. In this model, parties

adaptively search for new platforms in an attempt to increase the number of votes they receive in an election. We can use the model to explore the influence of the platform-selection process on the final equilibrium state achieved by the system.

Parties in our model are incrementally adaptive and incompletely informed. This contrasts with standard rational choice approaches in which parties are either completely informed optimizers (with knowledge of each voter's utility function) or act as Bayesians. In reality, practical considerations impose restrictions on a party's computational ability, the amount and type of information it has access to, and on the maximum allowable policy change it can make in any one election. For example, parties rely on imperfect and imprecise polling data to gather information about voters' preferences, and must restrict platform changes from year to year to maintain both credibility and a coherent organization.

From our perspective, no single model of party behavior is likely to be accurate. Computational modeling grants us the flexibility to consider various types of behavior and to learn which findings are particular to the assumptions and which are generic. We initially used three alternative adaptive algorithms: random search, hill-climbing, and genetic algorithms. In all of these methods parties attempt to improve the number of votes their platforms receive by adaptively generating new platforms. In random search, a party first generates a set of random new platforms constrained to be close to its current platform, and then adopts the best one. In hill-climbing (discussed in further detail below), a party generates a new random platform near its current platform, and keeps the better of the two (of course, this may imply that its current platform remains unchanged). This procedure is then iterated for a fixed number of steps. Finally, genetic algorithms (Holland, 1975) generate new platforms from a biased (by performance) set of old platforms via some simple platform formation operators (Kollman et al., 1992). Our findings suggest that few implications depend significantly on the heuristic used by parties, provided the parties are neither omniscient nor dim-witted. Thus, in the experiments presented here we only report the results from hill-climbing.

the voter may gain by choosing the party advocating the (*Yes, Yes, Yes*) platform.

Our actual hill-climbing procedure works as follows. A randomly generated *current platform* is given to each party prior to any elections. When a party is given a chance to adapt, it first generates a new platform by randomly perturbing its current platform on up to three issues. Thus, the new platform is constrained to lie in a neighborhood of the current one. If the new platform yields a higher vote total, it becomes the party's current platform; otherwise the current platform remains unchanged. This search process continues for eight iterations. Hill-climbing always bases its search on the current best-known platform.¹⁰ After the eighth iteration, another party is given a similar chance to adapt. After each party adapts, we return to the first party and begin a new cycle of adaptation. In the work reported here, five such cycles are allowed. At the end of these five cycles, an election is held in which each party proposes its current platform.

III. The Computational Model

In this section, we describe findings from computational experiments on Tiebout competition. Computational findings can be particular to the parameter values selected, so we have endeavored to test many sets of parameter values, and the results reported below appear to be robust to reasonable variations. For these experiments, we used 1,000 agents (citizens) and 11 binary issues.

A. Sequence of Events

The computational model begins by randomly creating preferences for each agent, and then randomly assigning each agent to a jurisdiction. Next, a series of ten *Tiebout cycles* is initiated. In each cycle, agents are first allowed to relocate. Following standard Tiebout models we assume that an agent moves to the jurisdiction providing her the highest utility.¹¹

¹⁰ Unlike random search, which first generates the set of new platforms (all based on the same current platform), and then selects the best one.

¹¹ If two or more jurisdictions have identical payoffs, the agent stays in her current location if it is one of these jurisdictions or, if not, moves to the jurisdiction with the

TABLE 1—SINGLE JURISDICTION: PER CAPITA UTILITY AFTER TEN ELECTIONS (200 TRIALS)

Institution	Per capita utility	(s.e.)
Democratic referenda	2.69	(0.12)
Direct competition (2 parties)	1.45	(0.13)
Direct competition (3 parties)	0.67	(0.13)
Direct competition (7 parties)	0.33	(0.13)
Proportional representation (3 parties)	1.33	(0.13)
Proportional representation (7 parties)	1.36	(0.13)

The next step in the cycle is to allow each jurisdiction to alter its platform. Under democratic referenda, each jurisdiction adopts the median position on each issue. With direct competition and proportional representation, parties alternate adapting their platforms via the hill-climbing algorithm discussed above.

We find that the configuration of agents and party platforms in our model typically settle down by the end of ten Tiebout cycles. Democratic referenda almost always settle into an equilibrium within about four cycles. The other institutions often achieve a steady state by the end of ten cycles, and even when this is not the case, they rarely show any significant increases in aggregate utility after this time.

The primary focus of the analysis is on the per capita utility of the citizens at the end of the tenth Tiebout cycle. Recall that each agent has an expected payoff of zero from a random platform and 100 from an ideal one. The standard errors reported in the tables are the standard errors of the estimate within a given cell.

B. Single-Jurisdiction Findings

The findings for a single-jurisdiction model are given in Table 1. Regardless of the institution, the final platforms provide significantly more utility than randomly generated platforms.

lowest "address" (each jurisdiction is given a unique address).

TABLE 2—MULTIPLE JURISDICTIONS: PER CAPITA UTILITY AFTER TEN ELECTIONS (200 TRIALS)

Institution	3		7		11	
	jurisdictions	(s.e.)	jurisdictions	(s.e.)	jurisdictions	(s.e.)
Democratic referenda	34.39	(0.15)	48.29	(0.13)	55.46	(0.12)
Direct competition (2 parties)	34.15	(0.14)	49.90	(0.13)	57.03	(0.13)
Proportional representation (3 parties)	35.56	(0.11)	51.80	(0.12)	58.93	(0.12)

Democratic referenda provide the highest aggregate utility, followed by two-party direct competition (and two-party proportional representation¹²), and seven- and three-party proportional representation. Democratic referenda result in median platforms, which, given our assumptions on the distribution of preferences, provide nearly maximal utility in a single jurisdiction. Under adaptive parties, the dynamics depend on the number of parties in the system. With only two parties, regions of the platform space that produce relatively high aggregate utility are quickly located by both parties. Once found, the parties tend to wander within these regions, often alternating which party carries the majority. Thus, we find relatively good performance under two-party institutions, with some diminution over democratic referenda due to the search process.

With more than two parties the dynamics are more complex. In these systems, parties initially tend toward a high utility region of the platform space, but once this occurs, single-party transients away from this region are common—if the other parties are closely clustered, then a single party may find increasing support if it moves away from the cluster and toward more extreme voters. The presence of transients and the likelihood that such extreme platforms will win increases with the number of parties in the system. Under direct competition this has a detrimental effect on aggregate utility, since the winning party implements its entire platform (this explains the low values observed under three- and seven-party

direct competition). Under proportional representation, however, the platform implemented by a jurisdiction usually does not belong to a single party, but rather is decided by weighted majority rule. This weighting process mitigates the impact of extreme parties. Note that as the number of parties increases under proportional representation, each agent is able to pick a party that advocates a platform near her own utility-maximizing platform, and thus proportional representation will begin to approximate democratic referenda.

C. Multiple Jurisdictions

With multiple jurisdictions agents can begin to sort according to their preferences. As long as jurisdictions are responsive to constituencies, aggregate utility should be nondecreasing in the number of jurisdictions, and will likely increase as multiple jurisdictions are better able to sort agents by preferences. Table 2 presents findings from models with 3, 7, and 11 jurisdictions.¹³ For all institutions considered, per capita utility increases with the number of jurisdictions. The addition of even a few jurisdictions allows a dramatic increase in utility. While the increase in utility was expected, what was not anticipated was that the *performance of the various political institutions completely reverses*. Democratic referenda, which were best in the one-jurisdiction model, are now worst, while proportional representa-

¹² The values for two-party proportional representation are not shown in the tables, but they are indistinguishable from those for two-party direct competition.

¹³ To simplify the subsequent discussion, we will only focus on the institutions of democratic referenda, two-party direct competition, and three-party proportional representation. These institutions are representative of the broader set of institutions used in the single-jurisdiction analysis.

TABLE 3—AGENT PREFERENCES

Agent	Issue 1	Issue 2	Issue 3
1	+1	+1	+1
2	+1	-1	+0.5
3	+1	+0.5	-1
4	+1	-1	-1
5	+1	-1	-1
6	-1	-0.5	-1
7	-1	+0.5	-1
8	-1	+0.5	-1

tion now performs best. With seven or more jurisdictions, three-party proportional representation is significantly better than two-party direct competition, which is significantly better than democratic referenda.

A full explanation of this performance reversal is left for the next section of the paper. Intuitively, the reversal is linked to the effectiveness of different institutions in sorting agents across jurisdictions. An institution like democratic referenda leads to very stable platforms and little sorting among the agents, while the instability of the platforms induced by proportional representation is such that low utility configurations of agents are easily disrupted. If platform instability is directly related to agent heterogeneity, then the system will naturally lead to configurations of high aggregate utility—configurations of agents that have relatively low aggregate utility are associated with high agent heterogeneity, and therefore are likely to lead to unstable platforms, which in turn cause agent migration. Eventually, as agents become better sorted, platform stability ensues, migration ceases, and equilibrium is achieved.

IV. The Power of Political Instability

In this section, we discuss how political instability may permit greater sorting of citizens and therefore be beneficial in a model of Tiebout competition. The argument hinges on the relationship between the level of political

instability within jurisdictions and the degree of heterogeneity of preferences at each jurisdiction. At present we do not have analytic proofs for many of these ideas. What follows is an informal explanation of our results, supported by evidence derived from the model. While mathematically derived results may be possible, such results may be difficult to obtain even in a much simplified model. We believe, however, that the lack of sufficient mathematical tools should not preclude analysis. In fact, results from our computational analysis should provide insights into how more formal proofs might proceed.

A. An Example

At the beginning of the paper we presented an example concerning alcohol policies in two small towns. Recall that policy instability in the system breaks the symmetry of an inferior equilibrium and results in an improved sorting of agents. Here we provide a more precise example that shows how two-party direct competition may be preferred to democratic referenda in a multiple-jurisdiction model.

In Table 3 we list the utility values for each of eight agents across three issues. Let a *Tiebout equilibrium with respect to an institution* exist when in each jurisdiction the current platform is consistent with the political institution and no agent wants to relocate (where consistency implies that the institution could have generated the observed platform given the agents in the jurisdiction). To simplify the discussion, we assume that for two-party direct competition any platform within the top-cycle set is consistent.¹⁴ Note that under this assumption, a platform in a jurisdiction may be overturned for two reasons: either it is defeated by another platform in the top-cycle set, or agent migration alters underlying voter preferences in the jurisdiction to such a degree that a new top-cycle set emerges.

First, consider a jurisdiction containing the first three agents. If the institution is

¹⁴ A platform belongs to the top-cycle set if it could be victorious over any other possible platform via some sequence of pairwise elections.

democratic referenda, then the platform will be (*Yes, Yes, Yes*), and the aggregate utility will be 4.0. In this jurisdiction, the platforms (*Yes, Yes, Yes*), (*Yes, Yes, No*), (*Yes, No, Yes*), and (*Yes, No, No*) are all members of the top-cycle set. The aggregate utilities from these platforms are 4.0, 3.5, 3.5, and 3.0 respectively. If we assume that the platform resulting from two-party direct competition is randomly chosen from this set, then democratic referenda are preferred to direct competition on utility grounds.

Now, suppose we add a second jurisdiction that contains agents 4–8. In this jurisdiction, democratic referenda will lead to (*No, No, No*). It is easy to show that no agent wants to relocate between the two jurisdictions under democratic referenda, and therefore this configuration is a Tiebout equilibrium with respect to this institution. A simple calculation shows that the aggregate utility in this equilibrium equals 4.0.

However, the above equilibrium is not necessarily stable under two-party direct competition. In two-party direct competition the platform (*Yes, Yes, Yes*) can be defeated by (*Yes, No, No*) in the first jurisdiction. Note that this new platform lowers the aggregate utility of the first group from 4.0 to 3.0. Under the new platform, agents 4 and 5 will now move from the second jurisdiction into the first, and (*Yes, No, No*) becomes the only member of the new top-cycle set in this jurisdiction. Without agents 4 and 5, (*No, Yes, No*) becomes the only member of the second jurisdiction's top-cycle set. It is straightforward to show that this new configuration of agents is a Tiebout equilibrium with respect to two-party direct competition (as well as democratic referenda). More importantly, the aggregate utility of this new configuration equals 5.5.

This example demonstrates how with multiple jurisdictions, the lack of a unique outcome from direct competition may induce sorting and increase aggregate utility. Like the example in the introduction of this paper, this increase in utility requires a temporary decrease of utility in a given jurisdiction in order to induce agent migration. However, the migration eventually leads to a higher aggregate utility for the overall system.

B. Insights from Simulated Annealing

To provide intuition for our computational results and the examples presented above, we apply insights from the literature on *simulated annealing*, a nonlinear optimization algorithm. Simulated annealing is a sequential search algorithm applied to a real-valued function, f . In simulated annealing, a neighborhood structure on the domain is first created. Each point, x , in f 's domain belongs to a neighborhood, $N(x)$, which contains at least one point different from x . Similar to hill-climbing, at each step simulated annealing randomly chooses a new point in the neighborhood of the status quo point and makes that new point the status quo if it has a higher value under f . Where simulated annealing differs from hill-climbing is that it may also move to new points that have lower values. This occurs with a probability that depends on the difference in the function values, $\Delta(x, \hat{x}) = f(x) - f(\hat{x})$, and a *temperature*, $T(t)$, which is a decreasing function of the time spent searching, t . Formally, if x is the status quo point and \hat{x} is a randomly chosen neighboring point of lower value, the probability of accepting this new point as the status quo is:

$$p(x, \hat{x}, t) = e^{-\Delta(x, \hat{x})/T(t)}.$$

Note that inferior points are more likely to be accepted when the temperature is high or the value of the inferior point is close to the status quo. If the difference in function values, $\Delta(x, \hat{x})$, is large relative to the temperature, $T(t)$, then the probability of acceptance is low. The temperature can be interpreted as the degree of leniency: high temperatures allow for almost any new point to be accepted, while low ones allow for a new point to be accepted only if the loss in value is small. The temperature decreases to zero according to an *annealing schedule*. As the temperature nears zero, only increasing moves are accepted, and search converges to a local optimum with respect to the neighborhood structure.

A substantial body of theory exists to explain the performance of simulated annealing. Bruce Hajek (1988) has shown that given any function there exists an annealing schedule

such that the simulated annealing procedure converges to the global optimum.¹⁵ To some extent these results are misleading, as the proofs, whether relying on Markov chain theory or real analysis, require that every point in the domain be evaluated. This obviously begs the question of why simulated annealing should be preferred over exhaustive search. In practice, even though simulated annealing often fails to locate the global optimum, it has been shown to be very effective at locating good solutions with relatively few searches.

A promising line of inquiry into the performance of simulated annealing analyzes the structure of the function's local optima. Consider search via a hill-climbing algorithm. Given a neighborhood structure, each local optimum has a basin of attraction defined by all those points that are located "down the hill" from the optimum. The larger a local optimum's basin of attraction, the more likely search ends at that optimum. Simulated annealing performs better on those functions with positive correlations between the values of local optima and the sizes of their associated basin of attraction. On such functions, local optima with relatively low values, and therefore small basins of attraction, are more likely to be rendered unstable by the noise from the annealing schedule. In effect, the mistakes due to the temperature smooth over the lesser-valued optima, and eventually, the high-valued optima—with large basins of attraction—become inescapable as the temperature tends to zero. In such spaces, simulated annealing performs "as if" it can recognize whether a local optimum's value was relatively high or low, escaping lower-valued optima in favor of higher-valued ones (Page, 1996).

In relating these insights to the performance of political institutions in a Tiebout model, we find strong, though not exact, connections. Consider a set of platforms across all jurisdictions, and the corresponding configuration of agents each attracts, as a point in the domain of the aggregate utility function. A neighborhood of such a point could consist of

all platforms (along with their corresponding reconfiguration of agents) that are located within a fixed distance (according to some metric) of the given set of platforms.

For explanatory purposes, we restrict the discussion to two political institutions: democratic referenda and two-party direct competition. Previously we have shown that democratic referenda yield a unique platform in each jurisdiction and that direct competition offers less stability since it may alternate platforms from among those available in the top-cycle set. We can interpret this instability as making mistakes, in the sense that the platforms chosen by direct competition typically have lower utility than the median platforms generated by democratic referenda. As was demonstrated above, these mistakes can be fortuitous in the long run, as they allow the system to break out of old equilibrium and to form new configurations of agents that promote the creation of new, utility-improving equilibria.

Of course, the presence of instability by itself is insufficient to lead to higher equilibria since it will disrupt, as well as find, good equilibria. Therefore, to account for the increase in performance we observe in our model, the instability must be systematically tied to the aggregate utility of the various equilibria.

We propose that, like simulated annealing, institutions that make more mistakes in relatively low-valued configurations will enable the system to act "as if" it recognizes the potential value of a local optimum and to escape inferior equilibria. Two characteristics of our model generate this fortuitous bias in error-making. First, aggregate utility for a configuration of agents is positively correlated with the homogeneity of preferences at each jurisdiction. Second, the instability of direct competition is linked to the heterogeneity of preferences in a jurisdiction.¹⁶ Combining these two effects, if agents in a configuration are not very homogeneous at each jurisdiction, aggregate utility will be low, and direct competition will generate new platforms

¹⁵ Similar results can be found in a special issue of *Algorithmica* (1991) dedicated to simulated annealing.

¹⁶ McKelvey (1986) has shown in a mathematical model that the size of the uncovered set decreases with the level of symmetry of preferences.

TABLE 4—AVERAGE NUMBER OF AGENT RELOCATIONS PER TRIAL (50 TRIALS)

Institution	3		7		11	
	jurisdictions	(s.e.)	jurisdictions	(s.e.)	jurisdictions	(s.e.)
Democratic referenda	864.16	(17.24)	863.2	(10.41)	887.3	(5.26)
Direct competition (2 parties)	915.68	(14.67)	1162.5	(8.63)	1293.7	(7.13)
Proportional representation (3 parties)	1277.60	(34.41)	1371.06	(25.75)	1420.28	(21.74)

which render the existing configuration unstable. If, however, the agents in a configuration are homogeneous at each jurisdiction, aggregate utility will be high and platforms will be stable. Thus, Tiebout equilibria with respect to direct competition are biased towards those platforms and agent configurations with higher aggregate utility.

To test these ideas, we turn to our computational model. If instability in the better performing institutions induces superior sorting, then we should observe significantly more relocations of agents under the better institutions. Table 4 shows the average number of agent relocations observed during each ten-cycle trial. As predicted, the performance of a given institution is closely tied to the number of relocations observed. Comparing Tables 4 and 2, relocations correlate positively with aggregate utility in multiple jurisdictions.

A more direct test of the sorting abilities of each institution is to measure the variance of the agent preferences within each jurisdiction or of the platforms across all jurisdictions. The former measurement indicates that democratic referenda result in the most preference variance, followed by two-party direct competition, and then three-party proportional representation. The results on platform variation are given in Table 5. In this table we report the average percentage of issues on which the platforms from two different jurisdictions differ (thus, if the platforms differ on exactly three issues, their value is $\frac{3}{11}$ or 28 percent). In all cases, proportional representation results in the most heterogeneous platforms across jurisdictions, followed by direct competition, and democratic referenda respectively. These rankings correspond with the rankings of aggregate utility and the number of relocations. More relocations and greater platform hetero-

geneity imply better sorting and, therefore, higher aggregate utility. Note that average platform heterogeneity decreases as the number of jurisdictions increases—this must occur since as the number of jurisdictions increases, the maximal possible average heterogeneity must decrease (two platforms may differ across all issues, but it is impossible for three platforms to each differ across all positions from the other two). In general, as the number of jurisdictions increases the average heterogeneity decreases as the space of possible platforms becomes more crowded.

Much of our argument as to performance differences among institutions requires that platforms need to be less stable as agent preferences become more heterogeneous. We test whether this in fact occurs by creating populations of agents with varying degrees of heterogeneity, and then measure the number of platform changes. To vary population heterogeneity, each agent's ideal point is made a convex combination of a *base preference*, common across all agents, and an *individual preference*:

$$(1 - \theta) \cdot \text{base} + \theta \cdot \text{individual},$$

where the individual and base preference were drawn from the same preference distribution used in the original model. We refer to θ as the *degree of heterogeneity*.

The results of a one-jurisdiction model with 250 agents are given in Table 6. The restriction to one jurisdiction guarantees that none of the changes in platforms are attributable to agent migration. We vary θ between 0.5 and 1.0. Values of θ below this range create agents whose preferences are too homogeneous: almost all have the same preferred platform. The findings are from 50 series of 10 elections, and

TABLE 5—AVERAGE DISTANCE BETWEEN PLATFORMS (50 TRIALS)

Institution	3		7		11	
	jurisdictions	(s.e.)	jurisdictions	(s.e.)	jurisdictions	(s.e.)
Democratic referenda	61.58	(0.84)	51.64	(0.32)	50.76	(0.23)
Direct competition (2 parties)	65.21	(0.41)	53.96	(0.23)	52.26	(0.13)
Proportional representation (3 parties)	66.67	(0.36)	56.12	(0.15)	53.32	(0.09)

flips equals the average number of issues that change on the platform during each election.

Democratic referenda have no flips across all degrees of heterogeneity since the lack of agent migration implies the identical median platform during all time periods. For the other institutions we find, as expected, that the average number of flips increases with the degree of heterogeneity. These findings strongly support our hypothesis that as preferences in a jurisdiction become less homogeneous the propensity for platform instability increases.

An additional test to support our annealing hypothesis is whether the introduction of an annealing process could improve the performance of democratic referenda. We create annealing in this situation by randomly forcing agents to move to suboptimal jurisdictions during each Tiebout cycle. Over time, we lower the probability of suboptimal moves to create an annealing effect. For example, we set the initial probability of moving to a suboptimal jurisdiction at 5 percent and decrease this probability by 5 percent each period. When we do this, the average utility significantly increases in almost all cases. Even though the utility increases, it does not achieve the level attained under the other two institutions—perhaps reflecting the crudeness of this process relative to mechanisms that are more responsive to the local conditions in each jurisdiction. Nonetheless, this experiment indicates that direct annealing can improve the outcome of democratic referenda.

The computational evidence strongly supports our hypotheses that: (1) annealing can improve aggregate utility in these systems by insuring better sorting of agents, and (2) that institutions vary in their ability to anneal the system. Although instability or noise may help break a system out of an inferior equilibrium,

it does not, in and of itself, guarantee that better equilibria will be selected. For this to occur, the noise must be reduced as the system converges on superior outcomes. While in general this may require some global knowledge of the objective function or some exogenous control factor, here decentralized institutions can achieve this end by linking instability to the degree of heterogeneity present in the local jurisdiction. Of course, this property was not explicitly designed into the institutions used here, but rather arises serendipitously.

V. Discussion

We find that in a model of Tiebout competition with multiple jurisdictions, some political institutions perform better than others by inducing better agent sorting and thereby generating higher aggregate utility. Ironically, these same institutions often perform relatively poorly in a single-jurisdiction model. The reason for this performance reversal is that multiple jurisdictions create a system with multiple equilibria. A minor mistake, which is harmful in the single-jurisdiction model, may be beneficial in the multiple-jurisdiction model since it can dislodge the system from a relatively bad local optimum, and induce agents to re-sort themselves into a better configuration.

Mistakes alone are not sufficient for finding and maintaining superior equilibria. In the multiple-jurisdiction model, institutions that generate high aggregate utility are those that create less stable platforms only when agents have locally more heterogeneous preferences. Since aggregate utility decreases as agent heterogeneity increases, this systematic instability induces more agent migration in low-valued configurations, yet simultaneously

TABLE 6—FLIPS AS A FUNCTION OF HETEROGENEITY
(50 TRIALS)

θ	Democratic referenda	Direct competition (2 parties)	Prop. rep. (3 parties)
0.50	0.00 (0.00)	0.24 (0.15)	0.69 (0.38)
0.55	0.00 (0.00)	0.21 (0.16)	0.80 (0.33)
0.60	0.00 (0.00)	0.23 (0.24)	0.80 (0.37)
0.65	0.00 (0.00)	0.26 (0.24)	0.77 (0.40)
0.70	0.00 (0.00)	0.31 (0.37)	0.88 (0.48)
0.75	0.00 (0.00)	0.38 (0.44)	1.01 (0.53)
0.80	0.00 (0.00)	0.53 (0.49)	1.26 (0.55)
0.85	0.00 (0.00)	0.68 (0.73)	1.94 (0.69)
0.90	0.00 (0.00)	1.37 (1.11)	2.84 (0.71)
0.95	0.00 (0.00)	2.65 (1.08)	3.78 (0.62)
1.00	0.00 (0.00)	3.36 (1.12)	4.02 (0.59)

allows the system to settle on high-valued ones. Such a process is consistent with the annealing argument presented in the previous section.

There are many simplifying assumptions in the current model. The parties lack policy preferences, voters are not strategic, and preferences are assumed to be linearly separable. If parties have policy preferences, then platforms may be even less representative than under the current model. Of course, even with less representative platforms, the relevant issue is whether the correlation of platform changes with the degree of heterogeneity of preferences is positive, and we see no reason why this would not be the case. The introduction of strategic voting may also influence the results. Clearly the existence of multiple equilibria and the complex dynamics in the model greatly complicate the task of voting strategically, and even with strategic voting there is no *prima facie* reason to suspect that the linkage between preference heterogeneity and policy instability will be lessened. In fact, this linkage may even increase for democratic referenda. If voters believe that they are in a configuration of low aggregate utility, they may be able to benefit by misrepresenting their preferences in order to force policy changes that spur relocations. Finally, experiments using nonlinear preferences generate similar results to those reported above.

The idea that decentralized mechanisms can be used to refine equilibrium outcomes toward those equilibria with higher aggregate payoff

has important implications for mechanism design. Our results indicate that three conditions help in the effective design of such mechanisms. First, the decentralized mechanism must be able to acquire information about the state of aggregate payoff from local information. In our model, agent heterogeneity at any given jurisdiction is a good proxy for the quality of global sorting. Second, the mechanism must be able to disrupt global equilibria through local action. For example, a platform change at a single jurisdiction might cause sufficient migration to result in a cascade of platform changes at other jurisdictions. Finally, the decentralized mechanism must link the local information to local action in such a way that as the global system improves, local action is dampened. Given these three conditions, a decentralized mechanism can bias the outcome towards those configurations of higher aggregate payoff.

The notion that institutions can act as natural annealing mechanisms may be useful in understanding the dynamic behavior and performance characteristics of many other systems. Tiebout models are just one example of a broad class of phenomena that must "sort" agents among alternative configurations with multiple equilibria. Other examples include traders pairing off in markets, laborers finding jobs, firms choosing production sites or product attributes, and individuals forming organizations. Under decentralized sorting mechanisms, these systems may get trapped in suboptimal configurations. If, however, there are means by which these poor configurations can be annealed in an appropriate manner, then the global system can escape these traps and achieve superior outcomes.

REFERENCES

- Algorithmica*. Ottawa, Canada: Springer-Verlag, 1991.
- Axelrod, Robert and Bennett, P. Scott. "A Landscape Theory of Aggregation." *British Journal of Political Science*, 1993, 23(2), pp. 211–33.
- Berglas, Eitan. "Distribution of Tastes and Skills and the Provision of Local Public Goods." *Journal of Public Economics*, November 1976, 6(4), pp. 409–23.

- Bewley, Truman F.** "A Critique of Tiebout's Theory of Local Public Expenditures." *Econometrica*, May 1981, 49(3), pp. 713-40.
- Cebula, Richard J.** "Interstate Migration and the Tiebout Hypothesis: An Analysis According to Race, Sex and Age." *Journal of the American Statistical Association*, December 1974, 69(348), pp. 876-79.
- Conley, John P. and Wooders, Myrna.** "Equivalence of the Core and Competitive Equilibrium in a Tiebout Economy with Crowding Types." *Journal of Urban Economics*, 1997, 41, pp. 421-40.
- DeVany, Arthur.** "Hard Cores, Soft Cores, and the Emergence of Self-Organized Coalitions." Working paper, University of California-Irvine, 1994.
- Dowding, Keith; John, Peter and Biggs, Stephen.** "Tiebout: A Survey of the Empirical Literature." *Urban Studies*, May 1994, 31(4-5), pp. 767-97.
- Durlauf, Steven N.** "Multiple Equilibria and Persistence in Aggregate Fluctuations." *American Economic Review*, May 1991 (*Papers and Proceedings*), 81(2), pp. 70-74.
- Fox, William F.; Herzog, Henry W. Jr. and Schlottmann, Alan M.** "Metropolitan Fiscal Structure and Migration." *Journal of Regional Science*, November 1989, 29(4), pp. 523-36.
- Hajek, Bruce.** "Cooling Schedules for Optimal Annealing." *Mathematics of Operations Research*, 1988, 13(4), pp. 331-60.
- Holland, John H.** *Adaptation in natural and artificial systems*. Ann Arbor, MI: University of Michigan Press, 1975.
- Holland, John H. and Miller, John H.** "Artificial Adaptive Agents and Economic Theory." *American Economic Review*, May 1991 (*Papers and Proceedings*), 81(2), pp. 365-71.
- Hoyt, William H.** "Local Government Inefficiency and the Tiebout Hypothesis: Does Competition among Municipalities Limit Local Government Inefficiency." *Southern Economic Journal*. October 1990, 57(2), pp. 481-96.
- Kollman, Ken; Miller, John H. and Page, Scott E.** "Adaptive Parties in Spatial Elections." *American Political Science Review*, 1992, 86(4), pp. 929-37.
- Konishi, Hideo.** "Voting with Ballots and Feet: Existence of Equilibria in a Local Public Good Economy." *Journal of Economic Theory*, February 1996, 68(2), pp. 480-509.
- Kramer, Gerald H.** "A Dynamical Model of Political Equilibrium." *Journal of Economic Theory*, December 1977, 16(2), pp. 310-34.
- McKelvey, Richard D.** "Intransitivities in Multidimensional Voting Models and Some Implications for Agenda Control." *Journal of Economic Theory*, June 1976, 12(3), pp. 472-82.
- _____. "Covering, Dominance, and the Institution-Free Properties of Social Choice." *American Journal of Political Science*, 1986, 30(2), pp. 283-314.
- Page, Scott E.** "Two Measures of Difficulty." *Economic Theory*, August 1996, 8(2), pp. 321-46.
- Percy, Stephen L.** "Revisiting Tiebout: Moving Rationales, Exiting Behavior, and Governmental Responses to Metropolitan Mobility." Presented at the Midwest Political Science Association, Chicago, Illinois, 1993.
- Percy, Stephen L. and Hawkins, Brett W.** "Further Tests of Individual-Level Propositions From the Tiebout Model." *Journal of Politics*, 1992, 54(4), pp. 1149-57.
- Peterson, Paul and Rom, Mark.** "American Federalism, Welfare Policy and Residential Choices." *American Political Science Review*, 1989, 83(3), pp. 711-28.
- Plott, Charles R.** "A Notion of Equilibrium and Its Possibility under Majority Rule." *American Economic Review*, September 1967, 57(4), pp. 787-806.
- Reschovsky, Andrew.** "Residential Choice and the Local Public Sector: An Alternative Test of the Tiebout Hypothesis." *Journal of Urban Economics*, October 1979, 6(4), pp. 501-20.
- Riker, William.** *Liberalism against Populism*. San Francisco: Freeman, 1982.
- Romer, Thomas and Rosenthal, Howard.** "Bureaucrats versus Voters: On the Political Economy of Resource Allocation by Direct Democracy." *Quarterly Journal of*

- Economics*, November 1979, 93(4), pp. 563–87.
- Rose-Ackerman, Susan.** “Market Models of Local Government: Exit, Voting, and the Land Market.” *Journal of Urban Economics*, 1979, 6(2), pp. 319–37.
- Samuelson, Paul.** “The Pure Theory of Public Expenditure.” *Review of Economics and Statistics*, 1954, 36(3), pp. 387–89.
- Schelling, Thomas C.** *Micromotives and macrobehavior*. New York: Norton, 1978.
- Sharp, E. B.** “Exit, Voice and Loyalty in the Context of Local Government Problems.” *Western Political Quarterly*, 1984, 37(1), pp. 67–83.
- Tiebout, Charles M.** “A Pure Theory of Local Expenditures.” *Journal of Political Economy*, October 1956, 64(3), pp. 416–24.
- Wooders, Myrna H.** “Large Games and Economies with Effective Small Groups,” in Jean-Francois Mertens and Sylvain Sorin, eds., *Game-theoretic methods in general equilibrium analysis*. Boston: Kluwer, 1994.
- Zax, Jeffrey S.** “Is There a Leviathan in Your Neighborhood?” *American Economic Review*, June 1989, 79(3), pp. 560–67.